

Performance Assessment Task				
Quilt Making Grade 4				
<p>The task challenges a student to demonstrate understanding of the concepts of 2-dimensional shapes and their properties. A student must be able to use the characteristics, properties, and relationships of two-dimensional geometric shapes in order to examine, compare, and analyze attributes of geometric figures. A student must be able to recognize which shapes have at least one right angle. A student must determine whether or not shapes have no lines of symmetry or at least one line of symmetry. A student must analyze 2-dimensional shapes and their properties and attributes to determine which shapes can and which shapes can't fit together without any gaps.</p>				
Common Core State Standards Math - Content Standards				
<p><b>Geometry</b>  <b>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</b>            4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.             4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</p>				
Common Core State Standards Math – Standards of Mathematical Practice				
<p><b>MP.6 Attend to precision.</b>            Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.</p> <p><b>MP.8 Look for and express regularity in repeated reasoning.</b>            Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation <math>(y - 2)/(x - 1) = 3</math>. Noticing the regularity in the way terms cancel when expanding <math>(x-1)(x+1)</math>, <math>(x-1)(x^2+x+1)</math>, and <math>(x-1)(x^3+x^2+x+1)</math> might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.</p>				
Assessment Results				
<p>This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.</p>				
Grade Level	Year	Total Points	Core Points	% At Standard
4	2008	8	4	46 %

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## Quilt Making

This problem gives you the chance to:

- work with 2D shapes and their properties
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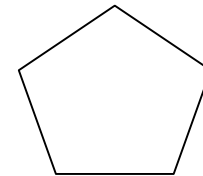
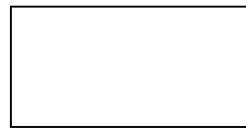
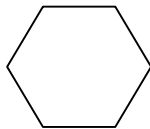
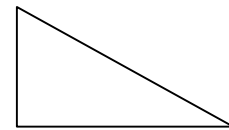
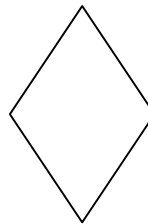
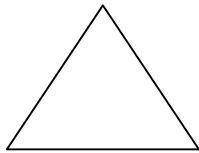
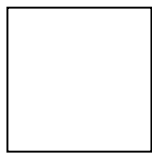
Matthew and his grandma make patchwork quilts.

Matthew helps his grandma sort the shapes.



1. Today his grandma wants shapes that have at least one right angle for her quilts.

Draw a ring around the shapes with at least one right angle.



2. The next quilt just needs shapes that have at least one line of symmetry.

Put a check mark (✓) inside the shapes that have at least one line of symmetry.

Name two shapes that **do not** have lines of symmetry.

\_\_\_\_\_

\_\_\_\_\_

Name three **quadrilaterals** that have lines of symmetry.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

3. Sometimes Matthew's grandma chooses to make a quilt using just one shape.

She can only do this using shapes that fit together.

Name one of the shapes shown above that will **not** fit together?

\_\_\_\_\_

Quilt Making	Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> <li>• work with 2D shapes and their properties</li> </ul> <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points
<p>1. Draws a ring around: <b>square, right triangle, rectangle</b>. All correct with no extras.</p> <p><i>Partial credit</i></p> <p>Two correct with no more than one extra.</p>	<p>2</p> <p>(1)</p>	<p>2</p>
<p>2. Puts a check mark inside the shapes: <b>square, equilateral triangle, rhombus, hexagon, rectangle and pentagon</b>. All 6 correct with no extras. For each extra deduct one point.</p> <p><i>Partial credit</i></p> <p>5-4 correct with no extras.</p> <p>Gives correct answers: <b>Parallelogram, right triangle</b> (accept scalene)</p> <p style="text-align: center;"><b>Square, rectangle, rhombus</b></p> <p>All 5 correct 3 points</p> <p><i>Partial credit</i></p> <p>4 correct</p> <p>3 or 2 correct</p>	<p>2</p> <p>(1)</p> <p>3</p> <p>(2)</p> <p>(1)</p>	<p>5</p>
<p>3. Gives correct answer: <b>pentagon</b>.</p>	<p>1</p>	<p>1</p>
<b>Total Points</b>		<b>8</b>

## Quilt Making

Work the task. Look at the rubric. What are the key mathematical ideas students need to have in order to be successful on this task?

Look at student work on part 1, identifying shapes with right angles. How many of your students:

Marked square, right triangle, & rectangle	Omitted the right triangle	Omitted the square	Omitted the rectangle	Circled the equilateral triangle	Circled the parallelogram	Circled the rhombus	Circled the hexagon

What activities have students done to help them understand and recognize right angles? Why do you think students had difficulty with this? How do different errors show different perceptions about the students' interpretations of "right angle"?

Look at student work for identifying shapes with lines of symmetry. How many of your students drew in lines of symmetry on the diagrams to show their thinking? \_\_\_\_\_

Why are students reluctant to draw or think on diagrams?

Now look at the choices students made for shapes with lines of symmetry. How many students:

Marked all 5 shapes	Marked no shapes	Marked the parallelogram	Forgot the rectangle	Forgot the equilateral triangle	Forgot the rhombus	Forgot the hexagon	Forgot the pentagon

Which errors might show that students only think about vertical lines of symmetry? Which errors might show that students only think about horizontal lines of symmetry? What other difficulties might lead to these errors?

How often do students in your class have opportunities to fold shapes to check for symmetry? Shapes, such as the parallelogram, seem symmetrical but folding shows that the two equal parts don't map onto each other.

Look at students' use of academic language for the second and third part of section 2. How many students:

- Used simply triangle instead of right triangle?
- Used diamond instead of rhombus?
- Drew in shapes or used descriptions rather than giving names to objects?

How do we help students develop academic vocabulary? What kinds of tasks give students opportunities to use academic language for a purpose?

How do you model academic language in your instruction? Do you try to weave in definitions with your academic language when teaching or giving students feedback?

Finally look at student work on part 3. How did your students do?

What opportunities do students have to build patterns or do tiling (tessellations)?

How does working with shapes in puzzles and other building activities help students develop their spatial visualization and help them work with and notice attributes like side length and angle size?

What other activities can help students develop these skills?

## Looking at Student Work on Quilt Making

Student A is able to meet all the demands of the task, including use of precise academic language.

### Student A

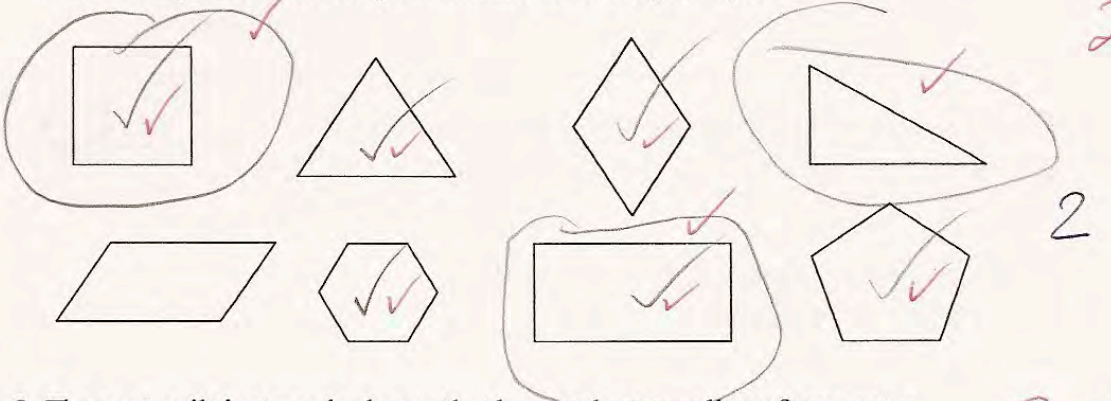
Matthew and his grandma make patchwork quilts.

Matthew helps his grandma sort the shapes.



1. Today his grandma wants shapes that have at least one right angle for her quilts.

Draw a ring around the shapes with at least one right angle.



2. The next quilt just needs shapes that have at least one line of symmetry.

Put a check mark (✓) inside the shapes that have at least one line of symmetry.

Name two shapes that **do not** have lines of symmetry.

Parallelogram ✓      Right Triangle ✓

Name three **quadrilaterals** that have lines of symmetry.

square ✓      rectangle ✓      rhombus ✓

3. Sometimes Matthew's grandma chooses to make a quilt using just one shape. She can only do this using shapes that fit together.

Name one of the shapes shown above that will **not** fit together?

pentagon ✓

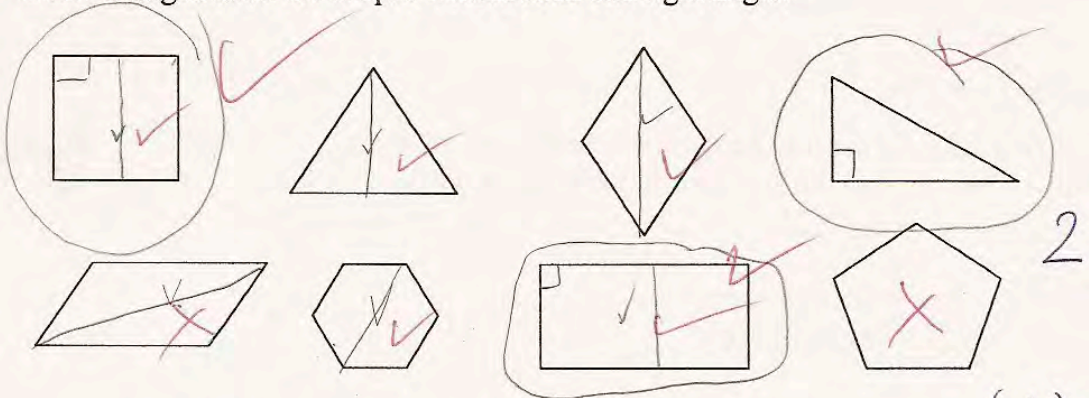
8

Student B understands what a right angle is and marks shapes with the symbol for right angle to assist in thinking about the task. Notice that the student also draws in lines of symmetry for the shapes. *How do we help students develop this habit of mind, using a diagram as a thinking tool?* Adding lines to diagrams becomes a critical skill in high school geometry, that few students have. Student B has the common misconception that because drawing a diagonal on a parallelogram creates equal size and shape pieces that it is symmetrical. *What type of activity would help this student see this error?* Notice that the student is not specific about the type of triangle that is not symmetrical. *How can we help students develop academic language?*

**Student B**

1. Today his grandma wants shapes that have at least one right angle for her quilts.

Draw a ring around the shapes with at least one right angle.



2. The next quilt just needs shapes that have at least one line of symmetry. (1)

Put a check mark (✓) inside the shapes that have at least one line of symmetry.

Name two shapes that **do not** have lines of symmetry.

pentagons X      triangles X

Name three **quadrilaterals** that have lines of symmetry.

rectangles ✓      diamonds X (1)  
squares ✓

3. Sometimes Matthew's grandma chooses to make a quilt using just one shape. She can only do this using shapes that fit together.

Name one of the shapes shown above that will **not** fit together?

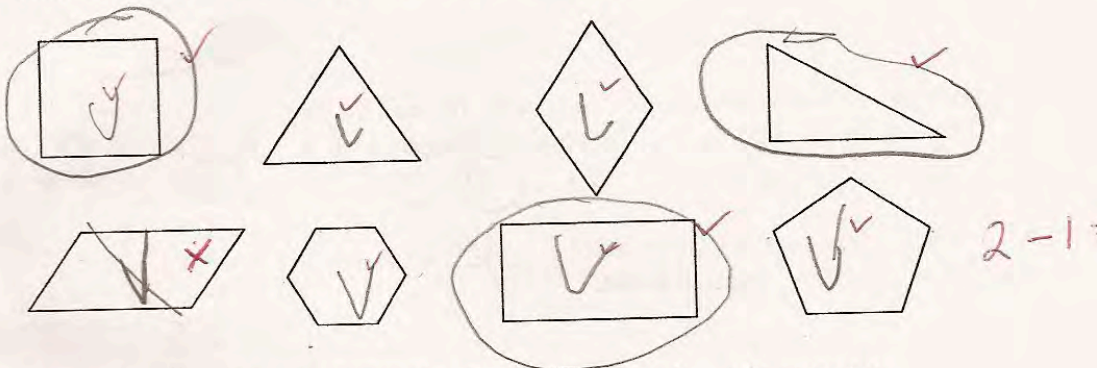
pentagons ✓ 8

Student C again thinks the diagonal of the parallelogram is a line of symmetry. The student is not specific about the type of triangle that is not symmetrical. The student does not think the diamond (rhombus) will fit together. *Why might the student think this? How might the student be interpreting "fit together"? What opportunities do students have to work with shapes to see for themselves how shapes fit together?*

**Student C**

1. Today his grandma wants shapes that have at least one right angle for her quilts.

Draw a ring around the shapes with at least one right angle.



2. The next quilt just needs shapes that have at least one line of symmetry.

Put a check mark (✓) inside the shapes that have at least one line of symmetry.

Name two shapes that **do not** have lines of symmetry.

triangle ✓ pentagon ✓

Name three **quadrilaterals** that have lines of symmetry.

square ✓ parallelogram ✓  
rectangle ✓

3. Sometimes Matthew's grandma chooses to make a quilt using just one shape.

She can only do this using shapes that fit together.

Name one of the shapes shown above that will **not** fit together?

Diamond ✓



Student D uses some lines to try and find the lines of symmetry. *What mistakes has the student made on the parallelogram?* The student does not have the academic language to identify the shapes and attempts to use descriptions instead. Notice that the student thinks that the right triangle is “upside down”. *Do students in your class get enough exposure to different orientations when working with shapes?*

**Student D**

1. Today my grandma wants shapes that have at least one right angle for her quilts.  
 Draw a ring around the shapes with at least one right angle.

2. The next quilt just needs shapes that have at least one line of symmetry.  
 Put a check mark (✓) inside the shapes that have at least one line of symmetry.  
 Name two shapes that **do not** have lines of symmetry.

the triangle on the right ×      an upside down of the triangle on the top right ×

Name three **quadrilaterals** that have lines of symmetry.

Square ✓      rectangle ✓      pentagon ×

3. Sometimes Matthew's grandma chooses to make a quilt using just one shape. She can only do this using shapes that fit together.  
 Name one of the shapes shown above that will **not** fit together?

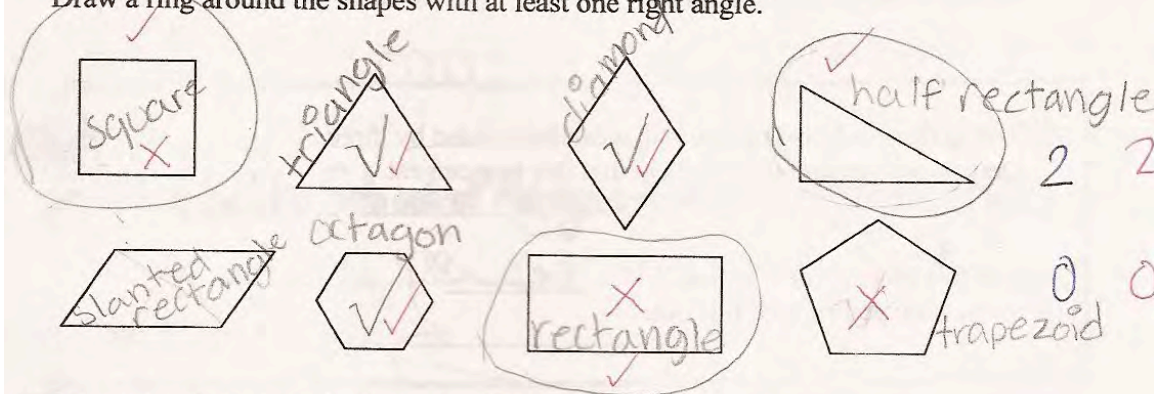
the triangle on the top right

Student E tries to name the shapes to help him think about the tasks in part 2 and 3. Notice that the student refers to the relationship between some shapes and a rectangle to compensate for lack of vocabulary. *What attributes might the student be paying attention to when confusing the pentagon for a trapezoid?* Notice that the student does not understand the term quadrilateral when trying to work on part 2.

**Student E**

1. Today his grandma wants shapes that have at least one right angle for her quilts.

Draw a ring around the shapes with at least one right angle.



2. The next quilt just needs shapes that have at least one line of symmetry.

Put a check mark (✓) inside the shapes that have at least one line of symmetry.

Name two shapes that **do not** have lines of symmetry.

trapezoid X      slanted rectangle X

Name three **quadrilaterals** that have lines of symmetry.

triangle X      diamond X  
octagon X

3. Sometimes Matthew's grandma chooses to make a quilt using just one shape.

She can only do this using shapes that fit together.

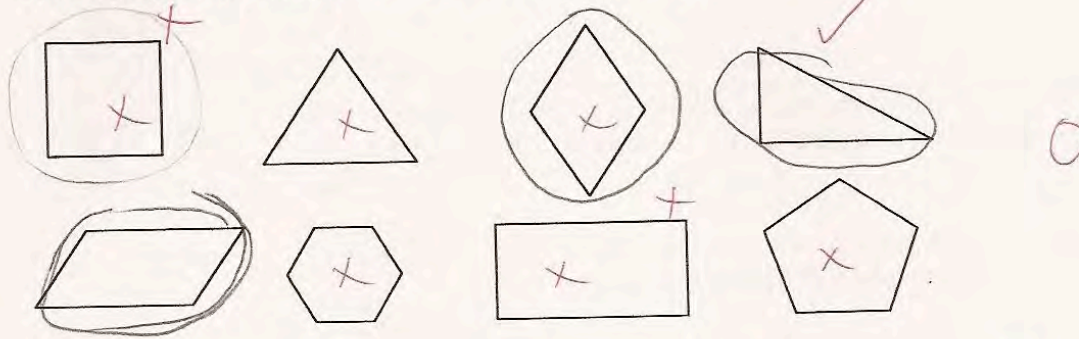
Name one of the shapes shown above that will **not** fit together?

half rectangle 0 0  
8

Student F does not understand what a right angle is. The student does not seem to understand line of symmetry, because the student does not see that the square has symmetry. *How would you help this student? What experiences will help the student?*

**Student F**

1. Today his grandma wants shapes that have at least one right angle for her quilts.  
Draw a ring around the shapes with at least one right angle.



2. The next quilt just needs shapes that have at least one line of symmetry.  
Put a check mark (✓) inside the shapes that have at least one line of symmetry.

Name two shapes that **do not** have lines of symmetry.

triangle ✓      square ✓

Name three **quadrilaterals** that have lines of symmetry.

Dime ✓      trapezoid ✓  
square ✓

3. Sometimes Matthew's grandma chooses to make a quilt using just one shape. She can only do this using shapes that fit together.

Name one of the shapes shown above that will **not** fit together?

not ✓

<b>Student Task</b>	Work with 2-dimensional shape and their properties.
<b>Core Idea 4 Geometry and Measurement</b>	<p><b>Use characteristics, properties, and relationships of two-dimensional geometric shapes. Examine, compare, and analyze attributes of geometric figures.</b></p> <ul style="list-style-type: none"> <li>• Classify 2-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles.</li> <li>• Understand line symmetry and predict the results of sliding, flipping or turning 2-dimesnional figures.</li> <li>• Investigate, describe, and reason about the results of combining and subdividing figures.</li> </ul>

*The mathematics of this task:*

- Identifying right triangles
- Finding shapes with lines of symmetry
- Understanding geometric terms: right triangle, rhombus, quadrilateral, etc.
- Understanding properties of angles, spatial visualization to see how shapes fit together

*Based on teacher observation, this is what fourth graders know and are able to do:*

- Identify right angles
- Name most of the shapes
- Find lines of symmetry

*Areas of difficulty for fourth graders:*

- Thinking the parallelogram has a diagonal or vertical line of symmetry
- Not having the vocabulary for rhombus and right triangle
- Not knowing the attributes of a quadrilateral
- Understanding tessellations, what shapes will or won't fit together

*Strategies used by successful students:*

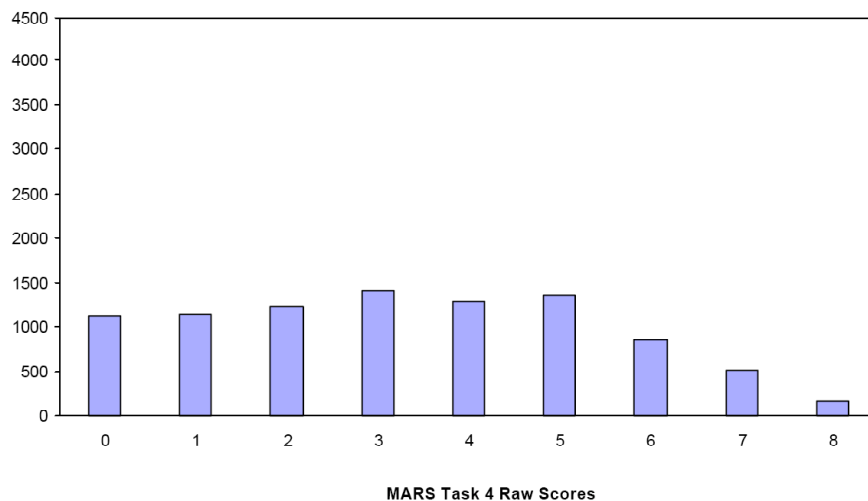
- Drawing in symbols for right triangles
- Drawing in lines of symmetry
- Naming the shapes before answering the questions in part 2 and 3
- Using diagrams as tools for thinking

Mean: 3.29      StdDev: 2.15

Table 23: Frequency Distribution of MARS Test Task 4, Grade 4

Task 4 Scores	Student Count	% at or below	% at or above
0	1127	12.4%	100.0%
1	1132	24.9%	87.6%
2	1222	38.4%	75.1%
3	1409	53.9%	61.6%
4	1294	68.2%	46.1%
5	1355	83.1%	31.8%
6	855	92.5%	16.9%
7	507	98.1%	7.5%
8	172	100.0%	1.9%

Figure 32: Bar Graph of MARS Test Task 4 Raw Scores, Grade 4



*The maximum score available for this task is 8 points.  
The minimum score for a level 3 response, meeting standards, is 4 points.*

Most students, 88%, could identify 2 or 3 shapes, using academic language, for quadrilaterals with lines of symmetry. Many students, 75%, could identify 2 shapes with right angles. Almost half the students, 46%, could find 3 shapes with right angles, identify 4 or 5 shapes with a line of symmetry, and name at least two quadrilaterals that are symmetrical. Less than 2% of the students could identify the two shapes that do not have lines of symmetry and correctly name the shape that does not tessellate. 12% of the students scored no points on this task. 71% of the students with this score attempted the task.

## Quilt Making

Points	Understandings	Misunderstandings
<b>0</b>	71% of the students with this score attempted the task.	Students did not understand the term quadrilateral. 16% of the students used diamond for rhombus. 30% named shapes that were not quadrilaterals.
<b>1</b>	Students could name 2 or 3 quadrilaterals that were symmetrical.	Students could not identify shapes with no line of symmetry. 11% thought a pentagon did not have symmetry, 11% thought a diamond did not have symmetry. 11% thought a hexagon did not have symmetry. 30% used the non-specific term triangle instead of right triangle.
<b>2</b>	Students could name shapes without symmetry and quadrilaterals with symmetry.	Students had difficulty with right angles. 16% thought the parallelogram had right angle. 9% thought the equilateral triangle had a right angle. 11% thought the hexagon had a right angle. 16% did not think a rectangle had a right angle. 11% thought a square did not have a right angle.
<b>4</b>	Students could identify shapes with right angles and name 4 shapes that either had no symmetry or quadrilaterals with symmetry.	Students struggled with identifying the shapes with at least one line of symmetry. 38% of the students thought the parallelogram had symmetry. 21% did not mark the pentagon as having symmetry. 18% did not mark the hexagon as having symmetry. 11% of the students did not mark the rectangle and 11% did not mark the square as having a line of symmetry.
<b>6</b>		Students could not name the triangle or the diamond in part 2 and could not identify the shape that wouldn't tessellate. 21% thought the right triangle would not tessellate. 11% thought "octagon", incorrect name for hexagon or pentagon, would not tessellate. 9% used the generic triangle as the shape that wouldn't fit together.
<b>8</b>	Students could identify shapes with right angles and lines or no lines of symmetry. Students knew the formal geometric terms for shapes, such as quadrilateral and rhombus. Students were able to visualize which shapes would not tessellate.	

## **Implications for Instruction**

Students need to have physical interaction with shapes to understand symmetry. They need experiences, such as paper folding, to see that parallelograms do not map when folded. Students need to work with shapes that have different lines of symmetry. Too often students only check for vertical or horizontal lines of symmetry, not both.

Students need opportunities to build designs with shapes or put together puzzles. These experiences help them to notice attributes like side length and properties of angles. Students also need to be comfortable drawing lines on diagrams to test for things like line symmetry. Using diagrams as tools for thinking helps them with the more complex diagrams in geometry, where lines need to be strategically added to diagrams to help with the logic and information needed to prove conjectures.

Sorting activities also help students focus on attributes of shapes as well as provide a reason for using academic vocabulary. Where possible, teachers should weave in definitions with the vocabulary in lessons and when responding to students ideas. Frequent repetition of definitions helps to build vocabulary.

## **Ideas for Action Research – The Logic of Sorting**

Sorting activities help students to focus on important attributes of shapes. Sorting also gives students reasons for using academic language. Consider two activities from the Virginia Department of Education website, What’s My Rule.

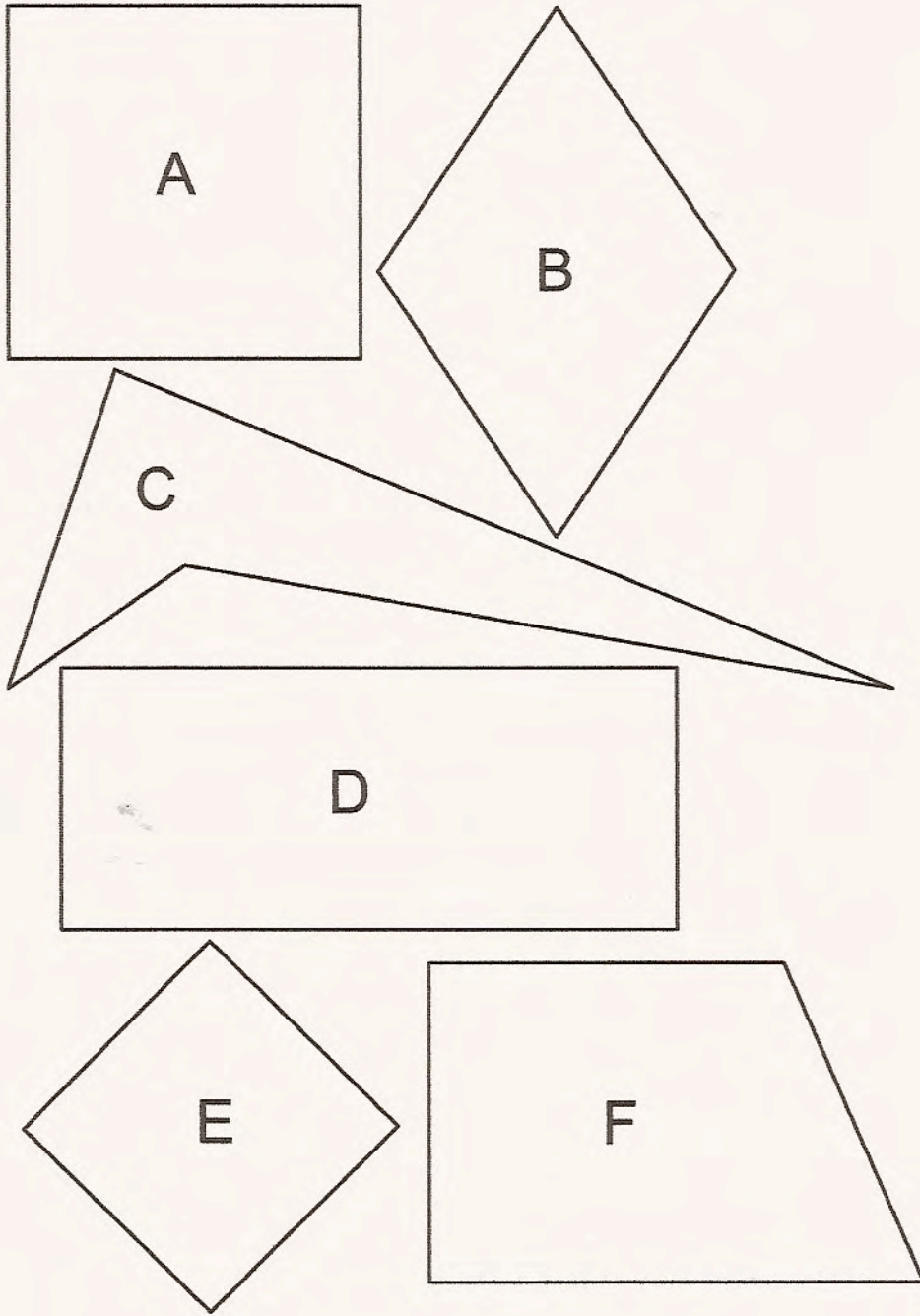
([www.doe.virginia.gov/VDOE/Instruction/Elem\\_M/geo\\_elem.html](http://www.doe.virginia.gov/VDOE/Instruction/Elem_M/geo_elem.html))

### **What’s My Rule**

1. Choose one player to be the sorter. The sorter writes down a “secret rule” to classify the set of quadrilaterals into two or more piles and uses that rule to slowly sort the pieces as the other players observe.
2. At any time, the players can call “stop” and guess the rule. The correct identification is worth 5 points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty.
3. After the correct rule identification, the player who figured out the rule becomes the sorter.
4. The winner is the first one to accumulate ten points.

A similar activity can be done with triangles.

## Quadrilateral Sorting Pieces





Quadrilateral Sorting Pieces  
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